

“No–Scalar–Hair” Theorems for Nonminimally Coupled Fields with Quartic Self–Interaction

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Self-gravitating scalar fields with nonminimal coupling to gravity and having a quartic self–interaction are considered in the domain of outer communications of a static black hole. It is shown that there is no value of the nonminimal coupling parameter ζ for which nontrivial static black hole solutions exist. This result establishes the correctness of Bekenstein “no–scalar–hair” conjecture for quartic self–interactions.

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I. INTRODUCTION

Black holes have no “hair” is the classical statement which summarizes the physics behind stationary configurations describing the final state of highly massive matter under gravitational collapse. The “no–hair” conjecture stresses the fact that short–range interactions decaying fast enough at infinity (“hair”), so that they have no contribution to Gauss–like conserved charges, are not allowed in the exterior of a stationary black hole. Hence, the black–hole classical degrees of freedom are restricted to those related to its conserved charges.

The accuracy of this conjecture was challenged in the last decade when several “hairy” black hole solutions were discovered (see Ref. [1] for a review and for the latest results see Ref. [2] and references therein). All these black holes shared together the feature of having nonlinear matter fields with no associated conserved charges.

It is believed that the conjecture remains true without reserve for other nonlinear fields. For example, a particular case of the conjecture concerning merely scalar fields has been formulated by Bekenstein and investigated by him and many other authors. The “no–scalar–hair” conjecture [3] asserts that for neutral scalar fields, nonminimally coupled to gravity, and with self–interaction potentials nontrivial black hole solutions are forbidden, i.e., any stationary black hole solution of the system must have a trivial scalar field (zero or constant) and consequently the solution must be given by one of the well–known vacuum black holes (Schwarzschild or Kerr). The aim of this work is to settle this conjecture for quartic self–interactions, i.e., to exclude static black hole hairs in the framework of nonminimally coupled to gravity quartic self–interacting scalar fields.

Scalar fields with minimal coupling and a quartic self–interaction were earlier studied on the background of Schwarzschild solution, exhibiting that only nonregular field configurations are allowed [4, 5]. Indeed, the present understanding of the topic excludes the existence

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of self-gravitating spherical scalar hairs with a general nonnegative self-interaction potential under minimal coupling [6, 7, 8, 9, 10].

Comprising nonminimal couplings and nonnegative self-interactions, hairy spherical black holes are inadmissible for nonminimal coupling parameters in the ranges $\zeta < 0$ and $\zeta \geq 1/2$ [3, 11]. Other restrictions on the nonminimal couplings have been established also in $n > 3$ dimensions [12], and even for $(2 + 1)$ -gravity [13]. More recently, numerical evidence against the existence of nontrivial black hole solutions for each value of the nonminimal coupling spectrum under consideration has been presented in Ref. [14].

In this work we analyze nonminimally coupled to gravity scalar fields with quartic self-interactions. The motivation for choosing a quartic self-interaction rests on the following: for massless scalar fields, several works on the subject [15, 16, 17, 18] reinforce the idea on the uniqueness of the nontrivial conformal Bekenstein black hole [19, 20]. Independently of a recent controversial debate on the existence of this black hole as a conventional exact solution to Einstein equations [21, 22], our view is that the oddities concerned with this solution may be due to the conformal invariance of the related model—actually, this solution was originally generated by a conformal transformation [20]. On the other hand, it is known that in four dimensions the relevant model remains conformally invariant under the inclusion of a quartic self-interaction. Hence, we decide to explore if conformal invariance also implies in this new case a nontrivial behavior in the presence of static black holes. Self-gravitation of these systems, including not only conformal coupling, is described by the action

$$S = \frac{1}{2} \int dv \left(\frac{1}{\kappa} R - \nabla_\mu \Phi \nabla^\mu \Phi - \frac{1}{2} \lambda \Phi^4 - \zeta R \Phi^2 \right), \quad (1)$$

where R stands for the scalar curvature, Φ is the scalar field, ζ is a parameter describing the nonminimal coupling between gravity and the scalar field, and λ is a self-coupling parameter characterizing the quartic self-interaction of the scalar field. We shall consider all the real values of the nonminimal coupling parameter ζ , on the other side the parameter λ is taken positive in order to the potential be nonnegative; a required condition for ruled out scalar hairs in the minimal model ($\zeta = 0$) [9].

The “no-scalar-hair” theorem we attempt to establish in this work consists in demonstrating that for all values of the nonminimal coupling parameter ζ , including the conformal one ($\zeta = 1/6$), any nontrivial behavior of the scalar field in presence of static asymptotically flat black holes is completely forbidden. That is, we shall study the scalar equation

$$\square \Phi - \lambda \Phi^3 - \zeta R \Phi = 0, \quad (2)$$

together with the Einstein equations

$$(1 - \kappa \zeta \Phi^2) R_\nu{}^\mu = \kappa \left[\nabla_\nu \Phi \nabla^\mu \Phi + \frac{1}{2} \delta_\nu{}^\mu \left(\frac{1}{2} \lambda \Phi^4 - \zeta \square \Phi^2 \right) - \zeta \nabla_\nu \nabla^\mu \Phi^2 \right], \quad (3)$$

arising both from action (1), not only in the case of the conformal coupling but for all the couplings permitted by the values of ζ , and we shall show that a scalar field fulfilling the above equations must vanish in the domain of outer communications of a static black hole.

In a static black hole, the stationary Killing field \mathbf{k} coincides with the null generator of the event horizon \mathcal{H}^+ in addition to being timelike and hypersurface orthogonal in all the domain of outer communications $\ll \mathcal{J} \gg$ (see Ref. [23] for sufficient conditions allowing that a stationary spacetime containing the above matter can be static). The simply connectedness

of $\ll \mathcal{I} \gg$ [24, 25], together with the above features, implies the existence of a global coordinate system, (t, x^i) , $i = 1, 2, 3$, in $\ll \mathcal{I} \gg$ [26] where $\mathbf{k} = \partial/\partial t$ and the metric can be expressed by

$$\mathbf{g} = -V dt^2 + \gamma_{ij} dx^i dx^j, \quad (4)$$

where V and γ are t -independent, γ is positive definite in all of $\ll \mathcal{I} \gg$, and V is a positive function tending to zero on \mathcal{H}^+ .

In the following section, it is shown that, in the domain of outer communications of a static asymptotically flat black hole, self-gravitating scalar fields with quartic self-interaction and nonminimally coupled to gravity, with nonminimal coupling parameter satisfying $\zeta < 0$ or $\zeta > 1/6$ vanish. Section III is devoted to establishing the result for $\zeta = 0$ and $\zeta = 1/6$, i.e., the minimal and conformal couplings respectively. In Sec. IV a similar result is established for the remaining values of the nonminimal coupling parameter within the interval $0 < \zeta < 1/6$. Finally, we provide some conclusions in the last Sec. V.

II. “NO-SCALAR-HAIR” THEOREM FOR $\zeta < 0$ AND $\zeta > 1/6$ COUPLINGS

In this section we analyze simultaneously the nonminimal couplings with $\zeta < 0$ as well as $\zeta > 1/6$; in both cases, it is possible to make the following redefinition for the scalar field $\varphi \equiv [6\kappa\zeta(\zeta - 1/6)]^{1/2} \Phi$. Upon contracting the Einstein equations (3) and inserting the scalar field equation (2) in the result, the following expression for the scalar curvature, in terms of the redefined scalar field, arises

$$-\zeta R = \frac{\nabla_\mu \varphi \nabla^\mu \varphi}{1 + \varphi^2} + \lambda b^2 \frac{\varphi^4}{1 + \varphi^2}, \quad (5)$$

where $b \equiv [6\kappa\zeta(\zeta - 1/6)]^{-1/2}$. The right-hand side of this expression contains nonnegative terms. In particular, the kinetic term can be written as $\nabla_\mu \varphi \nabla^\mu \varphi = \gamma_{ij} \nabla^i \varphi \nabla^j \varphi$ in the coordinates used in (4), where γ is positive definite. Two facts follow from this consideration, first, by asymptotic flatness ($R = 0$) the scalar field vanishes asymptotically due to vanishing of the second term at the right-hand side of (5). Second, since the domain of outer communications and the event horizon are regular regions, the left-hand side of (5) is bounded everywhere, and hence, from the bounded behavior of the second term on the right-hand side of (5), the scalar field is also bounded everywhere.

Inserting now the expression for R (5) in the scalar equation (2), in terms of φ , one arrives at

$$\square \varphi = \frac{1}{1 + \varphi^2} (-\varphi \nabla_\mu \varphi \nabla^\mu \varphi + \lambda b^2 \varphi^3). \quad (6)$$

By means of the above equation, we shall show that φ vanishes in all of $\ll \mathcal{I} \gg$, for a static asymptotically flat black hole.

Let $\mathcal{V} \subset \ll \mathcal{I} \gg$ be the open region bounded by the spacelike hypersurface Σ , the spacelike hypersurface Σ' , and the corresponding portions of the horizon \mathcal{H}^+ and the spatial infinity i^0 ; where the spacelike hypersurface Σ' is obtained by shifting each point of Σ by a unit value of the parameter along the integral curves of the stationary Killing field \mathbf{k} .

Multiplying the scalar equation (6) by φ , integrating by parts over \mathcal{V} , and using the Gauss theorem, one arrives at the following integral identity,

$$\left[\int_{\Sigma'} - \int_{\Sigma} + \int_{\mathcal{H}^+ \cap \overline{\mathcal{V}}} + \int_{i^0 \cap \overline{\mathcal{V}}} \right] \varphi \nabla^\mu \varphi d\Sigma_\mu = \int_{\mathcal{V}} \left(\frac{\nabla_\mu \varphi \nabla^\mu \varphi}{1 + \varphi^2} + \lambda b^2 \frac{\varphi^4}{1 + \varphi^2} \right) dv, \quad (7)$$

where $\overline{\mathcal{V}}$ stands for the closure of the set \mathcal{V} . On the left-hand side of (7) the boundary integral over the hypersurface Σ' cancels out that over Σ , since Σ' and Σ are isometric surfaces having inverted normals.

By asymptotic flatness ($R = 0$) the boundary integral over $i^o \cap \overline{\mathcal{V}}$ vanishes, since φ approaches asymptotically the zero value, see expression (5). In fact, for a stationary asymptotically flat spacetime scalar curvature behaves at infinity as $R = O(1/r^3)$, where $r^2 \equiv \sum_{i=1}^3 (x^i)^2$ is the radial coordinate related to asymptotically Euclidean coordinates at infinity. From relation (5) between the scalar curvature and the scalar field it follows that $\varphi = O(1/r^{3/4})$ at infinity in order that the existence of the scalar field is consequent with asymptotic flatness. Hence, the boundary integrand behaves as $\varphi \nabla_\mu \varphi = O(1/r^{5/2})$ and since the surface element goes as $d\Sigma^\mu = O(r^2)$, it can be concluded that the boundary integral vanishes appropriately at infinity.

We now shall prove that the boundary integral over the portion of the horizon $\mathcal{H}^+ \cap \overline{\mathcal{V}}$ also vanishes. To this end, we select the most natural volume 3-form at the horizon which is given by $\boldsymbol{\eta}_3 = -*\boldsymbol{n}$, where, if \boldsymbol{l} is the null generator of the horizon, \boldsymbol{n} is the other future-directed null vector ($n_\mu l^\mu = -1$) orthogonal to the spacelike cross sections of the horizon, and the star $*$ stands for the Hodge dual operator. It can be shown that with this volume 3-form the measure of integration on \mathcal{H}^+ is the standard one in this region [18, 27, 28]

$$d\Sigma_\mu = 2n_{[\mu} l_{\nu]} l^\nu d\sigma,$$

where $d\sigma$ is the surface element. With the above measure, the integrand at the horizon is written as

$$\varphi \nabla^\mu \varphi d\Sigma_\mu = \varphi [l_\mu \nabla^\mu \varphi + (l_\nu l^\nu) n_\mu \nabla^\mu \varphi] d\sigma. \quad (8)$$

The stationary Killing field \boldsymbol{k} of a static black hole coincides at the horizon with its null generator \boldsymbol{l} , hence, $l_\mu \nabla^\mu \varphi = \mathcal{L}_{\boldsymbol{k}} \varphi = 0$ by the staticity of the scalar field φ . Then, using also the bounded behavior of φ (5), the first term of the integrand (8), $\varphi l_\mu \nabla^\mu \varphi$, must vanish at the horizon. To show the vanishing at the horizon of the second term of the integrand (8), $\varphi (l_\nu l^\nu) n_\mu \nabla^\mu \varphi$, we shall introduce a null tetrad basis at the horizon (see also the approach of [18, 27] on this subject). This tetrad is composed by the null fields \boldsymbol{l} and \boldsymbol{n} , together with two linearly independent spacelike fields tangent to the spacelike cross sections of the horizon. Using this tetrad the metric is written at the horizon as

$$\boldsymbol{g} = -\boldsymbol{l} \otimes \boldsymbol{n} - \boldsymbol{n} \otimes \boldsymbol{l} + \boldsymbol{g}^\perp, \quad (9)$$

where \boldsymbol{g}^\perp stands for the metric projection on the spacelike cross sections of the horizon, which are orthogonal by definition to the null vectors \boldsymbol{l} and \boldsymbol{n} . Expanding the gradient of the scalar field $\nabla \varphi$ in the above base at the horizon, we obtain

$$\nabla \varphi = -(n_\mu \nabla^\mu \varphi) \left[\boldsymbol{l} + (l_\nu l^\nu) \boldsymbol{n} \right] + \nabla^\perp \varphi, \quad (10)$$

where orthogonality of the scalar gradient to the null generator of the horizon has been used, $l_\mu \nabla^\mu \varphi = 0$. Here again the notation ∇^\perp represents the projection of the gradient on the spacelike cross sections of the horizon, being orthogonal to \boldsymbol{l} and \boldsymbol{n} . Special attention must be given to the term involving the norm of the null generator; one has to keep it in the expansion, since there is no evidence *a priori* that this term vanishes. Using the expansion (10), we write the square of the scalar field gradient as

$$\nabla_\mu \varphi \nabla^\mu \varphi = -(l_\nu l^\nu) (n_\mu \nabla^\mu \varphi)^2 + \nabla_\mu^\perp \varphi \nabla^{\perp \mu} \varphi. \quad (11)$$

Inserting directly the expression of the metric in the given tetrad (9) within the square of the scalar field gradient, a different representation for this kinetic term can be obtained, i.e.,

$$\begin{aligned}\nabla_\mu \varphi \nabla^\mu \varphi &= g_{\mu\nu} \nabla^\mu \varphi \nabla^\nu \varphi \\ &= (-l_\mu n_\nu - n_\mu l_\nu + g_{\mu\nu}^\perp) \nabla^\mu \varphi \nabla^\nu \varphi \\ &= -2(l_\nu \nabla^\nu \varphi)(n_\mu \nabla^\mu \varphi) + \nabla_\mu^\perp \varphi \nabla^{\perp\mu} \varphi.\end{aligned}\tag{12}$$

Subtracting the expressions for the square of the gradient (11) and (12), one obtains

$$\left[(l_\nu l^\nu)(n_\mu \nabla^\mu \varphi) - 2(l_\mu \nabla^\mu \varphi) \right] (n_\mu \nabla^\mu \varphi) = 0.\tag{13}$$

The case in which $n_\mu \nabla^\mu \varphi = 0$ is trivial, since the product of this term times the norm of the null generator, $(l_\nu l^\nu) n_\mu \nabla^\mu \varphi$, automatically vanishes as well. The interesting case occurs when $n_\mu \nabla^\mu \varphi \neq 0$, under this circumstance the term within the brackets in (13) must vanish, which in turn implies, from the staticity of the scalar field $l_\mu \nabla^\mu \varphi = \mathcal{L}_k \varphi = 0$, that the term $(l_\nu l^\nu) n_\mu \nabla^\mu \varphi$ is vanishing. From these properties, using the bounded behavior of the scalar field (5), it follows that the second term of the integrand (8), $\varphi (l_\nu l^\nu) n_\mu \nabla^\mu \varphi$, also vanishes at the horizon.

With vanishing of the boundary integral at the portion of the horizon $\mathcal{H}^+ \cap \overline{\mathcal{V}}$, there remain no contributions on the left-hand side of (7), thus

$$\int_{\mathcal{V}} \left(\frac{\gamma_{ij} \nabla^i \varphi \nabla^j \varphi}{1 + \varphi^2} + \lambda b^2 \frac{\varphi^4}{1 + \varphi^2} \right) dv = 0,\tag{14}$$

using the coordinates of (4). Since γ is positive definite, the integrand above is non-negative, hence (14) implies vanishing of each term of this integrand in \mathcal{V} , from which it follows finally that φ vanishes in \mathcal{V} , and therefore in all of $\ll \mathcal{J} \gg$ by staticity.

In this way, the “no-scalar-hair” theorem has been proven for a field with a quartic self-interaction with $\zeta < 0$ or $\zeta > 1/6$ nonminimal couplings to the gravity of a static asymptotically flat black hole.

III. “NO-SCALAR-HAIR” THEOREM FOR $\zeta = 0$ AND $\zeta = 1/6$ COUPLINGS

This section deals with the minimal, $\zeta = 0$, and conformal, $\zeta = 1/6$, couplings. For the minimal case the results of references [6, 7, 8, 9, 10] are known, which apply to any non-negative self-interaction. For the conformal case there are still no results in the literature on the subject using self-interactions, except those of Ref. [29] for massive fields.

The reason for dealing with both couplings in the same fashion is that the corresponding scalar equation is the same in both cases. Actually, for the minimal coupling $\zeta = 0$ the scalar equation (2) reduces to

$$\square \Phi - \lambda \Phi^3 = 0.\tag{15}$$

The same equation also describes the conformal coupling $\zeta = 1/6$, due to the fact that the conformal symmetry implies $R = 0$; this can be noted from the contraction of the Einstein equations (3) for $\zeta = 1/6$, and the insertion of the scalar equation (2) into the result.

The only constant solution to equation (15) is the trivial one: $\Phi = 0$; hence, it is the only correct asymptotic solution in an asymptotically flat spacetime. We shall show for both

couplings $\zeta = 0$ and $\zeta = 1/6$ that if the scalar field Φ vanishes asymptotically, it must vanish in all of $\ll \mathcal{I} \gg$.

Multiplying equation (15) by $\tanh \Phi$ and integrating by parts over the previously defined region \mathcal{V} gives, after using the Gauss theorem,

$$\begin{aligned} & \left[\int_{\Sigma'} - \int_{\Sigma} + \int_{\mathcal{H}^+ \cap \overline{\mathcal{V}}} + \int_{i^o \cap \overline{\mathcal{V}}} \right] \tanh \Phi \nabla^\mu \Phi d\Sigma_\mu \\ &= \int_{\mathcal{V}} (\text{sech}^2 \Phi \nabla_\mu \Phi \nabla^\mu \Phi + \lambda \Phi^3 \tanh \Phi) dv. \end{aligned} \quad (16)$$

On the left-hand side of (16), the integrals over the hypersurfaces Σ' and Σ mutually cancel out again by isometry between both hypersurfaces, and by the opposite direction of its normals in the boundary integration. The integral over $i^o \cap \overline{\mathcal{V}}$ is zero because the scalar field Φ must vanish asymptotically (see the discussion in [27] on the fall-off rates for the scalar field following from the left-hand side of the Einstein equations in Bondi coordinates, such arguments are independent of if there exists a self-interaction potential or not). The remaining boundary integral over the portion of the horizon $\mathcal{H}^+ \cap \overline{\mathcal{V}}$ has again a vanishing integrand; using the same arguments as in the previous section it can be shown that the term $\nabla^\mu \Phi d\Sigma_\mu$ of the integrand vanishes at the horizon and since $\tanh \Phi$ is ever bounded, independent of the value of the scalar field at the horizon, the product of both terms, $\tanh \Phi \nabla^\mu \Phi d\Sigma_\mu$, vanishes at the horizon.

Therefore, the boundary integral in (16) vanishes and, in the coordinates of (4), we obtain

$$\int_{\mathcal{V}} (\text{sech}^2 \Phi \gamma_{ij} \nabla^i \Phi \nabla^j \Phi + \lambda \Phi^3 \tanh \Phi) dv = 0. \quad (17)$$

The above integral identity, together with the non-negativeness of the associated integrand terms within it, imply vanishing of the scalar field Φ in \mathcal{V} , and hence in all the domain of outer communications $\ll \mathcal{I} \gg$.

IV. “NO-SCALAR-HAIR” THEOREM FOR $0 < \zeta < 1/6$ COUPLINGS

At last, we shall concentrate on this section in the remaining interval, $0 < \zeta < 1/6$, of nonminimal couplings to gravity. The procedure is similar to that used in Sec. II, but demands a more careful treatment of these couplings. In this case it will be also useful to redefine the scalar field, $\varphi \equiv [6\kappa\zeta(1/6 - \zeta)]^{1/2} \Phi$ (notice the difference with the notation used in Sec. II).

The scalar curvature in the above interval of nonminimal couplings, obtained by substitution of the scalar equation (2) into the contracted Einstein equations (3), and with the above redefinition of the scalar field, is expressed by

$$\zeta R = \frac{\nabla_\mu \varphi \nabla^\mu \varphi}{1 - \varphi^2} + \lambda b^2 \frac{\varphi^4}{1 - \varphi^2}, \quad (18)$$

now $b \equiv [6\kappa\zeta(1/6 - \zeta)]^{-1/2}$. Each term on the right-hand side of this expression has the same sign again. Since the left-hand side of the above relation is bounded everywhere, this implies that each term at the right-hand side of (18) is bounded as well. In particular, the bounded character of the second term on the right-hand side of (18) guarantees that $\varphi^2 \neq 1$.

On the one hand, the domain of outer communications $\ll \mathcal{J} \gg$ is a connected set, hence, its image under the continuous function φ is also a connected set, which implies we have only one of the following two cases: or $\varphi^2 < 1$ in the whole of $\ll \mathcal{J} \gg$, or inversely $\varphi^2 > 1$ in all of $\ll \mathcal{J} \gg$. On the other hand, asymptotic flatness ($R = 0$ at infinity) demands that each term at the right-hand side of (18) vanishes asymptotically, in particular, vanishing of the second term implies that φ approaches the zero value at infinity, i.e., only the range

$$\varphi^2 < 1, \quad (19)$$

is allowed for the scalar field in the domain of outer communications $\ll \mathcal{J} \gg$ of a static asymptotically flat black hole.

Substituting the scalar curvature from (18) into the scalar equation (2), one obtains

$$\square\varphi = \frac{\varphi}{1-\varphi^2} (\nabla_\mu\varphi\nabla^\mu\varphi + \lambda b^2\varphi^2). \quad (20)$$

The rest of the proof of vanishing of the scalar field φ in all of $\ll \mathcal{J} \gg$ resembles the proof previously accomplished in Sec. II. Multiplying the scalar equation (20) by φ , and integrating by parts in the volume \mathcal{V} using the Gauss theorem yields this time the integral identity

$$\left[\int_{\Sigma'} - \int_{\Sigma} + \int_{\mathcal{H}^+ \cap \overline{\mathcal{V}}} + \int_{i^0 \cap \overline{\mathcal{V}}} \right] \varphi \nabla^\mu \varphi d\Sigma_\mu = \int_{\mathcal{V}} \left(\frac{\nabla_\mu \varphi \nabla^\mu \varphi}{1-\varphi^2} + \lambda b^2 \frac{\varphi^4}{1-\varphi^2} \right) dv. \quad (21)$$

The boundary integral in (21) is identical to the one appearing in Sec. II, and φ satisfies similar properties to those satisfied by the redefined scalar field of the just mentioned section, i.e., φ vanishes asymptotically with suitable fall-off, and it is a bounded function at the horizon (19). All of the above implies that there are no nonzero contributions in the boundary integral on the left-hand side of (21), from which one concludes that

$$\int_{\mathcal{V}} \left(\frac{\gamma_{ij} \nabla^i \varphi \nabla^j \varphi}{1-\varphi^2} + \lambda b^2 \frac{\varphi^4}{1-\varphi^2} \right) dv = 0, \quad (22)$$

in the coordinates of (4). Due to the condition (19) and since γ is positive definite, each term in the integrand above is non-negative. Hence, vanishing of the volume integral implies vanishing of each one of these terms, and in particular, vanishing of the scalar field φ in the volume \mathcal{V} , result which can be extended to the rest of the domain of outer communications $\ll \mathcal{J} \gg$ by the staticity of the scalar field.

V. CONCLUSIONS

We have shown the following “no-scalar-hair” theorem: for any value of the nonminimal coupling parameter ζ , a self-gravitating scalar field with quartic self-interaction and non-minimally coupled to gravity unavoidably vanishes in the domain of outer communications of a static asymptotically flat black hole. In particular, for the conformal coupling $\zeta = 1/6$ the conformal invariance of the system has no relevance in the existence of nontrivial black hole solutions, as it would be the case in the absence of self-interaction. The established results completely settle the Bekenstein “no-scalar-hair” conjecture [3], on the behavior of scalar systems in presence of black holes, for quartic self-interactions and any value of the nonminimal coupling parameter. With a vanishing scalar field the right-hand side of the Einstein equations (3) is zero, and the black hole associated to the system is necessarily the Schwarzschild one.

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